Chapter 8

PRELIMINARY VEHICLE STRUCTURE DESIGN APPLICATION

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Abstract The Method of Imprecision, or M_oI, is a formal method for incorporating imprecise information into a design process. This methodology has been exercised on a problem in preliminary vehicle structure design in collaboration with VW Wolfsburg. Results show that the method is useful in trading off multiple conflicting attributes, including styling preferences and engineering requirements.

Keywords:
Industrial Applications of DTM; Vehicle Structure Design; Design Methods and Models; Design Representations; Computational Methods of Design; Fuzzy Sets

Introduction

Preliminary design is inherently imprecise [3, 4, 12, 40], and many preliminary design decisions are made informally. Preliminary design has enormous economic importance, as much of the cost of a design is determined by these (often informal) preliminary decisions [37]. A further complication is the difficulty of communicating imprecise information between different members or groups involved in the design process. Many "interface" decisions are made after design analysis is complete; these *post hoc* decisions can result in costly redesigns.

The Method of Imprecision, or M_oI [39, 1] has been developed to formally incorporate imprecise information into the engineering design process. In the summer of 1997, an application of the M_oI to preliminary vehicle structure design was demonstrated for Volkswagen Wolfsburg. The application serves both to demonstrate the capabilities of the method, and as an introduction to some of the underlying concepts.

A brief introduction to the Method of Imprecision is followed by a description of the demonstration project. The application of the MoI to the problem is then described in detail, and the implications of the results are discussed.

The Method of Imprecision

This introduction is necessarily brief; the application serves as a further tutorial to explicate the ideas reviewed here.

The original work on the MoI [38, 39] formulated the design problem as a decision problem: given a set of candidate designs, identified by vectors $\vec{d}$ of *design variables* in a Design Variable Space (*DVS*), a set of performances, described by vectors $\vec{p}$ of *performance variables* in a Performance Variable Space (*PVS*), and a mapping $f : \vec{d} \mapsto \vec{p}$, choose the candidate design $\vec{d}^*$ which maps to the "best" possible performance $\vec{p}^* = f(\vec{d}^*)$. So stated, there is insufficient information to determine what constitutes a "best" performance, and hence a "best" design. On the one hand, requirements are imprecise, while on the other hand, there is no obvious way to compare different performance variables which are usually not expressed in the same units.

The need to include imprecision in engineering design can be illustrated by a simple example. Figure 8.1 shows a specification for one performance variable (p_j), with the *performance preference* μ_p on the vertical axis. As specifications are commonly written, $p_j \geq 500$ km would be represented by the dashed line (the sharp-edged rectangular step), where $\mu_p = 1$ in the acceptable region, and $\mu_p = 0$ for unacceptable values. However, this crisp specification (or requirement) indicates that two different designs, one with $d_j = 500 - \epsilon$, and another with $d_j = 500 + \epsilon$, would have completely different acceptabilities, no matter how small ϵ becomes. Thus two designs, indistinguishably different in d_j (as $\epsilon \rightarrow 0$), have completely different preferences: one is completely acceptable and one is unacceptable. This situation makes no sense.

Alternatively, the solid line shown in Figure 8.1 indicates a smooth transition of acceptability of performances from unacceptable ($\mu_p = 0$) to acceptable ($\mu_p = 1$), and thus reflects a more realistic specification. The range over which the transition from unacceptable performance to most desired performance takes place will depend on the particular design problem, and may be more or less steep, and smooth or faceted.

Thus the MoI introduces the notion of *preferences*, denoted by $\mu \in [0, 1]$, both to represent the imprecision inherent in the preliminary design problem, and to provide a basis for comparison between different attributes. Performance preferences μ_p on the *PVS* express the requirements more completely

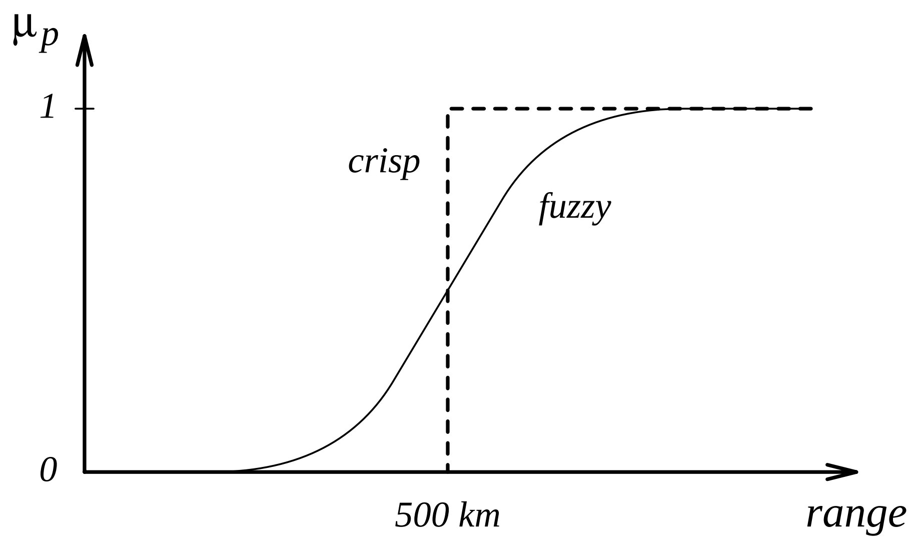

Figure 8.1 Example Imprecise Specification.

than crisp targets. In addition, engineers express *design preferences* μ_d on design variables, allowing the incorporation of performance aspects that are not explicitly calculated by f. Preferences are naturally represented and manipulated using the mathematics of fuzzy sets [42].

The design preferences $\mu_d(\vec{d})$, which are specified on the *DVS*, can be mapped onto the *PVS* by use of the *extension principle* [41]. The various preferences are then combined with an aggregation function $\mathcal{P}$; at first, the M$_o$I made use of two different aggregation operators [24], the non-compensating $\mathcal{P}_{\min}(\mu_1, \mu_2) = \min(\mu_1, \mu_2)$ for situations where the overall performance is dictated by the lowest-performing attribute, and the compensating $\mathcal{P}_{\Pi}(\mu_1, \mu_2) = \sqrt{\mu_1 \mu_2}$, when high performance on one attribute is deemed to partly compensate for lower performance on another. Each candidate design $\vec{d}$ thus has an associated overall preference:

$$\mu_o(\vec{d}) = \mathcal{P}\left(\mu_d(\vec{d}), \mu_p(f(\vec{d}))\right)$$

(where $\mu_d(\vec{d})$ and $\mu_p(\vec{p})$ are themselves aggregations of their constituent preferences), and candidate designs can be compared on the basis of this overall preference.

Further research on the M$_o$I developed techniques for including noise [26] and adjustments, or *tuning parameters* [25], in the imprecision calculations, and placed an axiomatic framework on the calculations [23]. Implementation of the M$_o$I continued with the development of a computational tool [19]. The applicability of the method was seen to be limited by large computational requirements, so the inclusion of Design of Experiments (DOE) approximations [21] and other computational innovations [18] followed. The need for more than two aggregation functions to model different trade-off levels was recognized, and a family of aggregations introduced [32, 33]. The M$_o$I is reviewed in more detail, and compared to other methods, in [1].

Other researchers have applied fuzzy methods to design optimization problems [7, 8, 28, 29, 6]. Related research includes: chemical process synthesis [9]; fuzzy constraint propagation applied to manufacturing [10]; fuzzy scheduling [11]; application of fuzzy methods to windturbine design [13]; multiobjective scheduling [14]; management of uncertain knowledge in engineering design [15]; engineering design optimization [16]; imprecise calculations in engineering design [5]; evaluation of design alternatives [17]; fuzzy MADM methods in system design [22]; fuzzy evaluations [27]; multiobjective fuzzy optimization [30, 34]; fuzzy ratings and utility analysis in preliminary design evaluation of multiple attributes [35]; scheduling system design [36]; and fuzzy multi-criteria decision making [44, 45, 43]. In addition, there is increasing interest in related work in evolutionary algorithms [46], including the combination of evolutionary algorithms with the M$_o$I [31].

Preliminary Vehicle Structure Design

The general vehicle structure design problem is the engineering of a *body-in-white*, which consists of the (usually metal) frame to which components and exterior panels are fastened. While there are interesting alternative solutions such as space frames and monocoques, this paper is concerned with the welded metal structure typical of passenger automobiles of the present day (see Figures 8.2 and 8.3). The vehicle structure engineers must design a body-in-white that meets certain measurable engineering targets such as stiffnesses, stress levels under load, and weight. In addition, they must satisfy many performance targets associated with less easily measured concepts such as style, manufacturability, and requirements of other engineering groups involved in the design process. These unmeasured peformances are handled informally, often by negotiation between groups working on the same vehicle. The M$_o$I was developed to allow for a formal approach to the incorporation of this imprecise information.

In order to avoid any difficulties involving confidential information, it was decided that an older model vehicle would provide an effective demonstration of the method. To this end a 1980 VW Rabbit (see Figure 8.2) was acquired. The vehicle was stripped to the structural body-in-white, and torsional and bending stiffnesses were measured. The intact body-in-white was found to

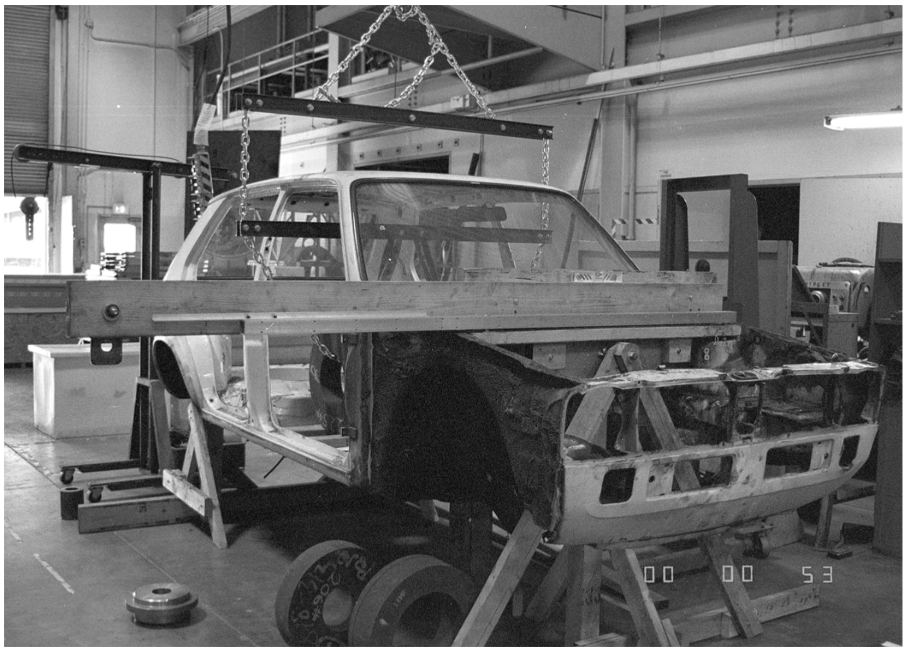

Figure 8.2 1980 VW Rabbit in Stiffness Testing

have a torsional stiffness of approximately 4900 N-m/degree and a bending stiffness of approximately 2500 N/mm. Tables of data from some of the load tests are shown in the Appendix. In addition, geometric data were gathered and used to create a solid model (Figure 8.3). The solid model and the structural stiffness information together were used to create and calibrate a finite element model (Figure 8.4).

The finite element model was parameterized with five[1] design variables:

1. A-pillar thickness (mm)
2. B-pillar thickness (mm)
3. floor pan thickness (mm)
4. floor rail thickness (mm)
5. B-pillar location (mm aft of a nominal point chosen by stylists)

and the performance was assessed with three measures:

[1]The demonstration here was conducted using a subset of the design variables; the method can be applied directly to a larger set of variables.

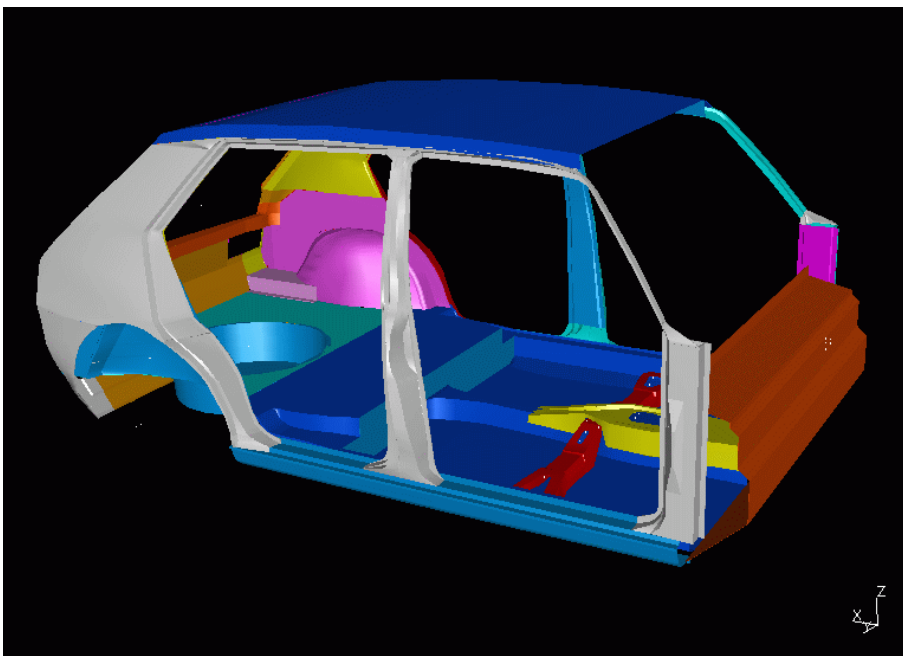

Figure 8.3 Geometric Model of Body-In-White in SDRC I-Deas

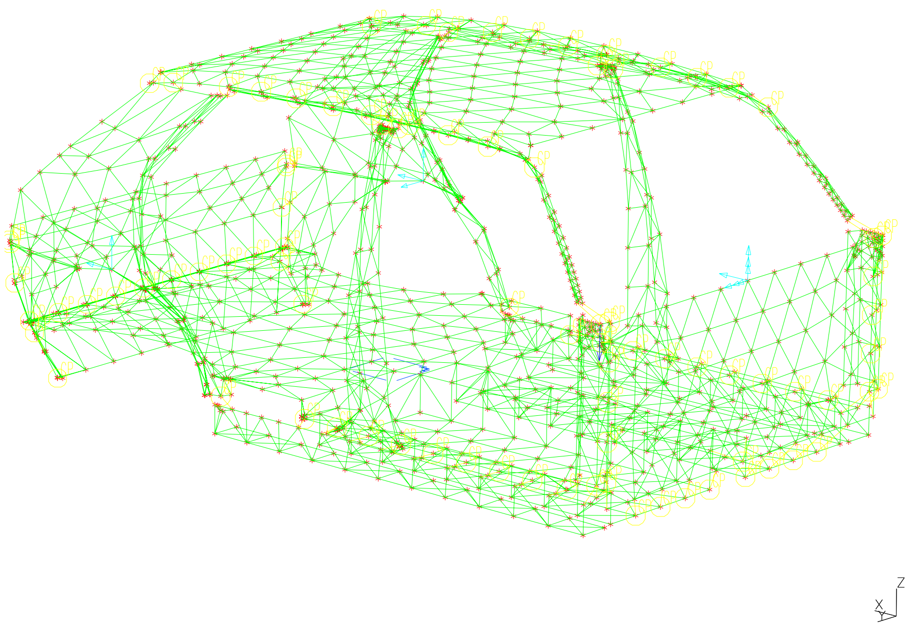

Figure 8.4 Finite Element Model of Body-In-White

1. Bending stiffness (N/mm)

2. Torsional stiffness (N-m/deg)

3. Weight (kg)

The stated design problem was to achieve 10% improvements over the reference model in the three measured performances. In addition, it was understood that that the design must not be difficult to manufacture, and that this year's model should have a somewhat longer and sleeker look.

Applying The $M_o I$ To Include Imprecise Information

While standard optimization methods could be used to determine the highest achievable bending stiffness, the highest achievable torsional stiffness, or the lowest achievable weight for this analysis model, such an optimization would not tell the designer which designs are the most promising when other relevant considerations are taken into account. On the one hand, there is a necessary trade-off between the stiffnesses and the weight; it is impossible to optimize both simultaneously. Additionally, there is other (imprecise) information to consider when making the decision, such as manufacturing and styling concerns. The application of the $M_o I$ to this problem involves constructing a differ-

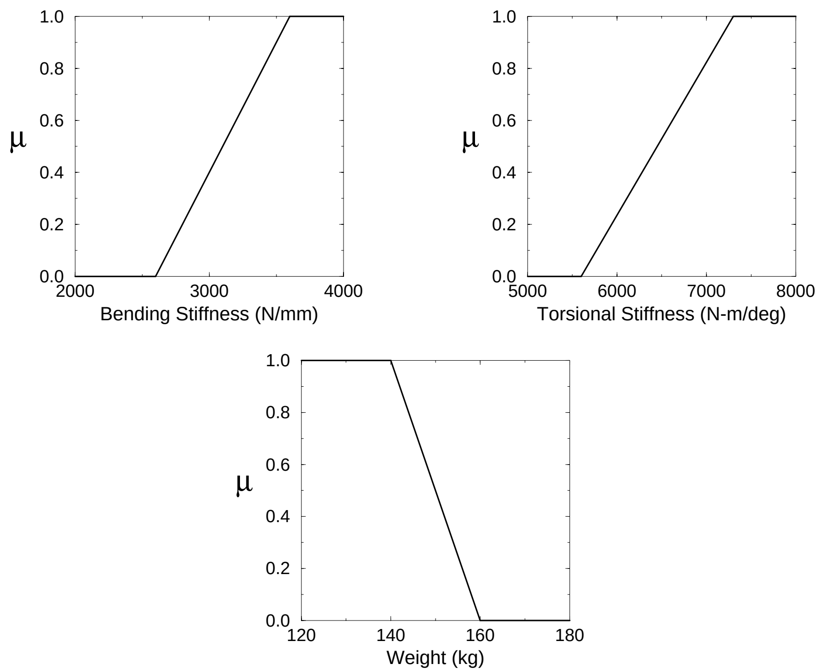

Figure 8.5 Imprecise Performance Requirements

ent "optimization" problem that includes the imprecise information that would be left to the negotiation stage in traditional design.

The calculated performance requirements on bending stiffness, torsional stiffness, and weight were originally expressed as targets of 10% improvements over the reference model. As was discussed above, this is unrealistically, and indeed unproductively, precise. In place of these hard targets, imprecise performance requirements were specified with a linear interpolation between two points. In the implementation of the M_oI, it is common to name the customer as the source of the performance preferences; in fact, it is more likely to be a manager, perhaps informed by market research, serving as the customer's proxy. To specify these imprecise requirements, the manager must answer two simple questions: "What is the lowest performance you can live with (where is $\mu = 0$)? What performance would satisfy you completely (where is $\mu = 1$)?" These bounds are clearly dependent on a number of factors, including the target market and the performance of competitors' products; we have found that engineering managers can answer these two questions with little more effort than is needed to settle on the initial crisp target. Figure 8.5 shows the imprecise requirements on stiffnesses and weight.

To include requirements on manufacturing, availability, style, and other things which are not calculated in the finite element analysis, designer preferences are specified on the design variables. As with the imprecise performance requirements, they range from $\mu = 0$ at the unacceptable limit to $\mu = 1$ at the most preferred. A preference is defined on each of the five design variables, as shown in Figure 8.6. Each preference is representative of imprecise information that can be incorporated using the M_oI:

1. The sheet steel for stamping the A-pillar is only available in certain increments, so this plot is discrete rather than continuous. The manufacturing engineer has a higher preference for thinner sheets, since they are easier to form; this is a design preference for manufacturability.

2. The B-pillar thickness is continuous and more complicated than the linear performance preferences. This preference does *not* indicate that the physical B-pillar might be 1.113 or 1.114 mm thick; rather it means that the designer knows that the finite element model is simplified, and that a high number for B-pillar thickness means that more reinforcing features will need to be added to the B-pillar. The designer would like to keep the B-pillar as simple as possible.

3. The floor pan thickness is preferred thicker by the designer for ease of attachments and for durability.

4. The floor rail thickness preference is an example of a sourcing, or availability preference; it states that some thicknesses are more easily obtained than others.

5. The design preference for B-pillar location comes from the stylists, and captures the directive for a longer, sleeker look for this year's model. It has been specified differently from the other design preferences, using α-*cuts* [1], so that the stylists have given a range of perfectly acceptable values, a range of barely acceptable values, and a range of values that fall in the middle. This method of specifying preferences can have computational advantages.

In addition to these preferences, each attribute is assigned a weight indicating its relative importance, and the way in which attributes trade-off against each other must also be specified. In this test example, it was determined that bending and torsional stiffness traded-off in a non-compensating manner — the lowest preference is maximized. Together they traded-off with weight in a compensating manner, so that high performance on stiffness could partly make up for low performance on weight, and vice versa. The designer preferences all traded-off in a compensating manner as well, with the styling preference

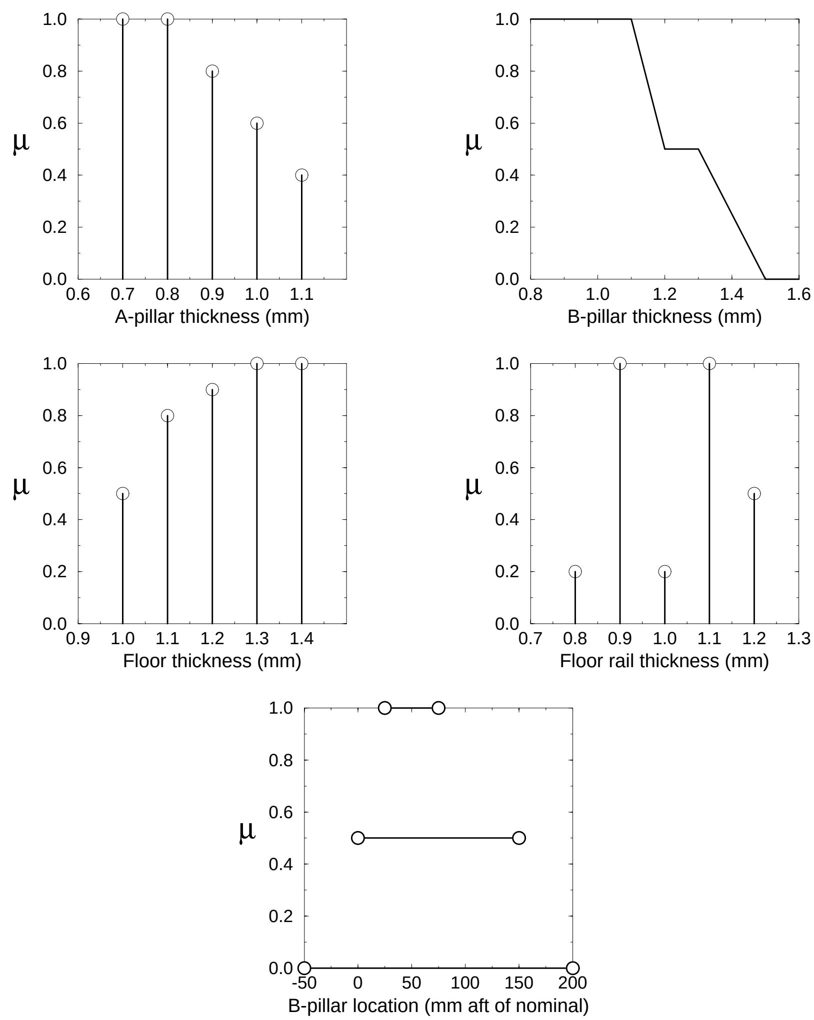

Figure 8.6 Designer Preferences

assigned a relatively high weight to reflect the importance of styling considerations in automobile design. The preferences for the computed performance variables are also weighted heavily. The correct determination of weights and trade-off strategies is crucial to the method, and a full range of strategies [33], of which the original compensating and non-compensating strategies are only two examples, is available.

Through the application of the M$_O$I, the design problem has been reformulated to be the maximization of the overall preference:

$$\begin{aligned} \mu_o(\vec{d}) &= \mathcal{P}\left[\mu_d(\vec{d}), \mu_p(f(\vec{d}))\right] \\ &= \mathcal{P}_\Pi\left[\mathcal{P}_\Pi\left(\mu_p(f_3(\vec{d})), \mathcal{P}_{\min}\left[\mu_p(f_1(\vec{d})), \mu_p(f_2(\vec{d}))\right]\right),\right. \\ &\quad \left. \mathcal{P}_\Pi\left(\mathcal{P}_\Pi\left[\mu_d(d_1), \mu_d(d_2), \mu_d(d_3), \mu_d(d_4)\right], \mu_d(d_5)\right)\right] \end{aligned}$$

The computation of $\mu_o(\vec{d})$ for a single design point $\vec{d}$ is limited by the finite element stiffness calculation, which takes about a minute on a Sun Ultra1-170MHz workstation; the calculations of weight and preference aggregation are of negligible cost, regardless of the trade-off strategies employed. Even in this relatively modest problem, where there are only five design dimensions, an exhaustive calculation of preferences over the design space is prohibitively expensive. The M$_O$I exploits the structure of the problem to speed the search for preferred solutions: the internal calculations linearize where possible, effectively reducing the dimension of the search space, and employ Powell's method to locate internal extrema [20].

Results

The design problem, including all imprecise information, was solved in two different ways. First, in order to demonstrate the method, the finite element analysis was run 3125 ($= 5^5$) times to provide a coarse but complete check of the entire design space. The point of peak overall preference of $\mu_o = 0.50$ was found at $\vec{d} = (1.0, 0.9, 0.9, 1.0, 50)$, where the design preferences μ_d are $(0.6, 1.0, 1.0, 0.5, 1.0)$; the stiffnesses and weight at this point were $\vec{p} = (2832, 5836, 147)$, with preferences $(0.23, 0.14, 0.62)$. The maximum achievable stiffnesses are 3365 N/mm ($\mu_p = 0.77$) in bending and 6029 N-m/degree ($\mu_p = 0.25$) in torsion, but the corresponding weight of 170 kg is unacceptable. Similarly, a weight of 144 kg ($\mu_p = 0.78$) is achievable, but stiffnesses drop to 2803 N/mm ($\mu_p = 0.20$) and 5730 N-m/degree ($\mu_p = 0.08$). The combined overall preference μ_o also takes into account the design preferences on style, manufacturability, and the like.

The power of the method lies not in an ability to find a single overall "best" point, but in the information it contains of how the total combined preference μ_o varies with each of the design variables. Although it is impossible to display all five dimensions varying at once, a tool was written that uses a commercial package (Matlab) to display results interactively. Using the tool, the designer can see the change in preference that would occur by varying each design variable independently from a chosen beginning point. Results can be seen on five simultaneous plots in two dimensions (see Figure 8.7), or on a three-dimensional surface plot (see Figure 8.8) with the remaining design variables set to nominal values. The interpretation of these graphical results is discussed in greater detail below.

Approximations

Naturally, the exhaustive evaluation of points in the design space would not be performed on a real design problem. It was performed here *only* for comparison purposes. An approach that utilizes Design of Experiments (DOE) [2] to approximate the finite element calculations for bending and torsional stiffnesses reached substantially similar results in only 21 runs (or approximately 20 minutes). The average difference (from the exhaustive evaluation) in bending stiffness was approximately 1%, with a maximum difference of less than 4%, while the average and maximum differences for torsional stiffness were both less than 1%.

In some cases, the nonlinearities of the analysis function f will defeat a linear or even polynomial approximation, but in many cases, such as the example presented here, these simple approximations can drastically reduce the required computation. Since precise answers are not required for preliminary design, it is sensible to exploit approximation tools when possible. If more computation can be justified, a more thorough calculation can be made.

Discussion

Engineering analysis usually requires some judgement on the part of the designer. Unless a full-scale exact prototype is to be built and tested, the accuracy of any calculated performance measure depends on the fidelity of the model employed. Even when exact data are available for some attributes, final decisions about a design incorporate other, unmodelled concerns, such as manufacturing and styling.

The MoI constructs a model of the entire decision process, expressing the calculated overall performance $\mu_o(\vec{d})$ as a function of the design variables. It depends on many factors: the function f for calculating measurable performances, the specification of design preferences μ_d and performance preferences μ_p, the weighting of these preferences, and the specification of trade-off aggregations between attributes. A change in any of these will affect the shape of the function μ_o in design space, and thus affect the decision. The analysis f is here relatively expensive to compute, and changes in the finite element

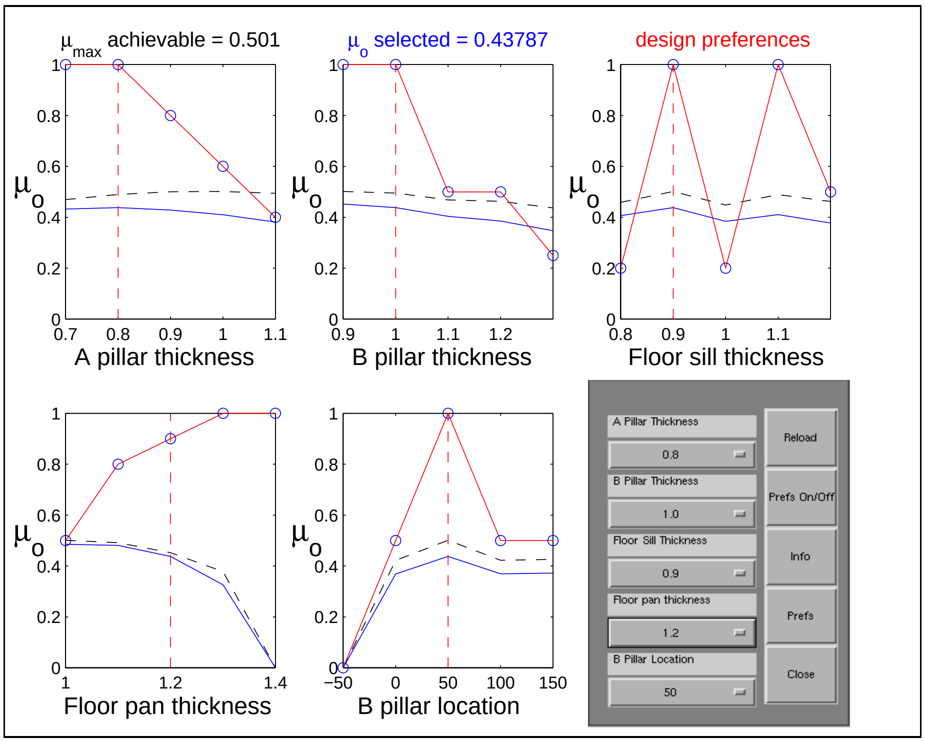

Figure 8.7 Graphical User Interface for Preference Display

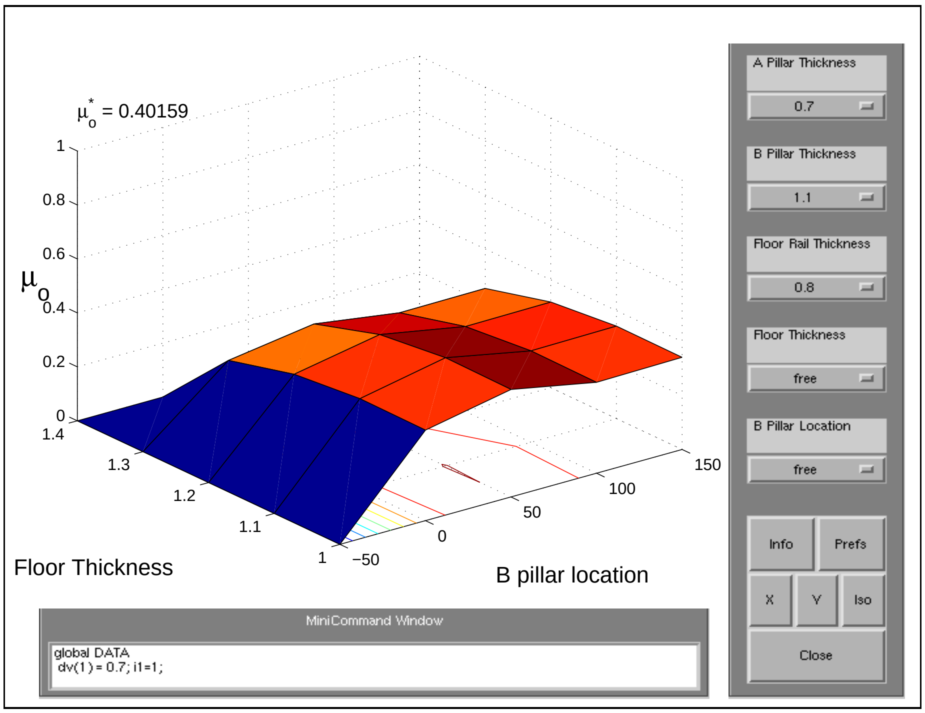

Figure 8.8 3-D Graphical User Interface for Preference Display

model are costly to propagate to overall preference. Changes in the other factors, on the other hand, are easily incorporated, as finite element results $f(\vec{d})$ are stored so that the same design point need never be analyzed more than once. This allows the M_OI to support an iterative decision process, when the information from the first round of calculations inspires a change in the preference structure.

In the example shown in this paper, the shape of the function $\mu_o(\vec{d})$ is sensitive to changes in the styling preference $\mu_d(d_5)$, which is not surprising, since this preference is accorded a large weight. This and other features of the design problem can be seen in the advanced interface shown in Figure 8.7. The vertical dashed (red) lines indicate the selected values of the design variables. In this figure, $\vec{d} = (0.8, 1, 0.9, 1.2, 50)$, a point which is representative, not optimal. The solid (blue) lines indicate how the overall preference μ_o would change by varying that design variable while holding the other four fixed at their current values. For instance, decreasing the floor pan thickness d_4 will result in a more preferred design. The dashed (black) lines show the maximum achievable μ_o for each value of each design variable. Finally, the solid (red) lines joining the circles are the specified preferences on design variables.

In this example, the desired improvements in performance were achieved by small changes to the sheet-metal thicknesses and B-pillar location. Visually the improved structure would appear quite similar to that shown in Figure 8.3. No change in vehicle structure configuration was required here.

Figure 8.7 shows that the overall preference μ_o varies qualitatively, though not quantitatively, with four of the five design variables; the exception is d_4, floor pan thickness. The variation of the overall preference μ_o with respect to d_4 shows a conflict between the calculated stiffness and weight requirements μ_p and the provisions for attachment and durability captured by the designer preference $\mu_d(d_4)$. This indicates d_4 as a likely candidate for change in a potential redesign. The resolution of the conflict would be achieved by a choice of d_4 that provides the best overall trade-off between the competing attributes. Designers can interact with the preference display to examine trends in the structure of the overall preference.

Conclusions

In preliminary vehicle structure design, as in preliminary engineering design in general, many important decisions are made informally on the basis of imprecise information. Concerns of styling and manufacturability, for instance, can carry great weight in the design process although they are not modelled by any formal analysis. The M_OI is a tool to formally incorporate such imprecise information into the design process, and thus to make decisions on a sound basis. In a demonstration of the M_OI prepared for VW Wolfsburg, concerns of manufacturing, styling, parts availability, and design were incorporated with the engineering analysis of the structural stiffness of a VW Rabbit. The results show the usefulness of the method in trading off these conflicting attributes.

Any analysis involving more than two design variables must contend with two difficulties, the exploding need for computation, and the problems of displaying results in several dimensions. The M_oI uses approximations, when feasible, to address the first difficulty, and an interactive graphical tool for preference display was developed and applied here to address the second.

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A Appendix: Stiffness Test Results

Torsion

Load (N)	Moment (N-m)	Deflection (mm)	Twist (deg)
0.00	0.00	0.00	0.00000
126.99	212.84	1.09	0.04407
275.78	455.03	2.41	0.09736
404.77	667.87	3.48	0.14041
551.55	910.06	4.47	0.18038
680.54	1122.90	5.72	0.23060
845.12	1394.45	6.99	0.28184
974.11	1607.28	8.08	0.32591

Fitting to $y = mx + c$:

$$m = 4960.74 \text{ N-m/deg}$$
$$c = -10.16 \text{ N-m}$$

Fitting to $y = mx + 0$:

$$m = 4917.04\text{N-m/deg}$$

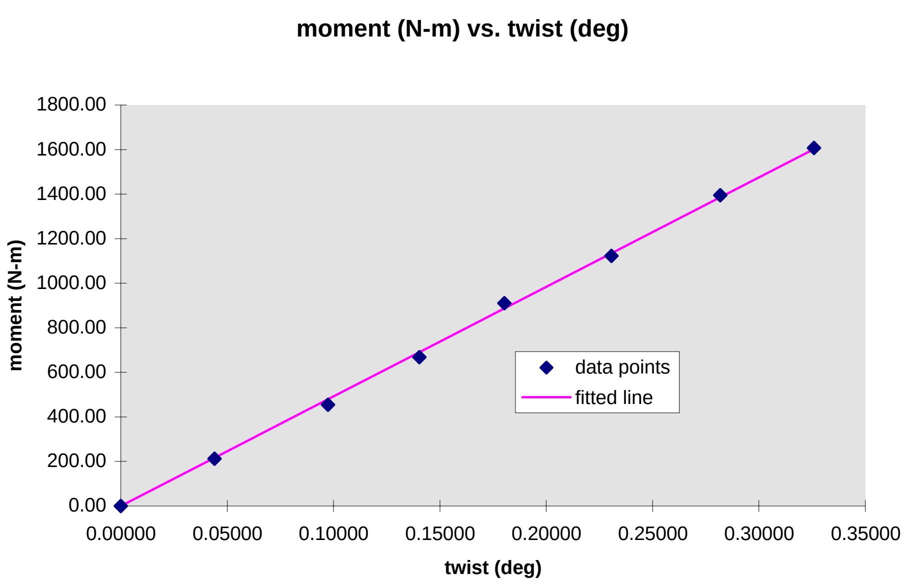

Figure A.1 Load Test, Torsional Stiffness

Bending

Load (N)	Deflection (mm)
0.00	0.00000
284.67	0.101060
551.55	0.17780
836.22	0.33020
2001.60	0.78740
2286.27	0.91440
2837.82	1.16840

Fitting to $y = mx + c$:

$$\begin{aligned} m &= 2426.16\text{N/mm} \\ c &= 51.79\text{N} \end{aligned}$$

Fitting to $y = mx + 0$:

$$m = 2484.80\text{N/mm}$$

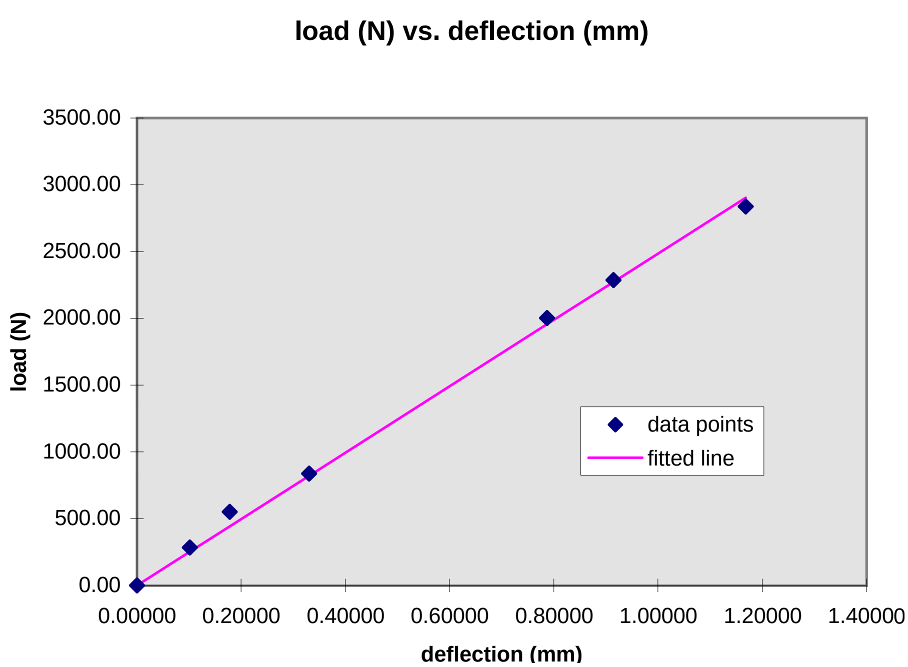

Figure A.2 Load Test, Torsional Stiffness