Imprecision in Engineering Design*

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ABSTRACT: The decisions with the greatest importance and potential cost (if wrong) are made early in the engineering design process. A method for representing and manipulating imprecise and vague information in design is described, particularly focused on the preliminary phase when the (fuzzy) imprecision and uncertainty in the descriptions of the design artifact are high. The preferences of designers and customers are captured with fuzzy sets. Formal methods for including noise, trade-off strategies and design iteration are included. Increasing the information available to a designer will reduce the risk of making design decisions incorrectly. Providing (fuzzy) set-based information to engineers can facilitate concurrency in design.

INTRODUCTION

Engineering design, the process of creating a new device to perform a desired function, intrinsically involves imprecision. This imprecision arises from the nature of the design problem, where one or more concepts are refined into a final design. At the concept stage, the designs are only vaguely, or imprecisely, described. Once the design process is complete, the design is described sufficiently precisely that it can be manufactured.

During the process of the refinement of a concept into a finished design, most information describing the design will be imprecise to some degree, reflecting the degree to which the designer has made final design decisions and refinements. To facilitate solving engineering design problems, and to assist design refinement, advanced design methods must represent and manipulate this intrinsic design imprecision.

A method, utilizing the mathematics of fuzzy sets, has been developed over the past 15 years at Caltech. This "Method of Imprecision" ($\mathrm{M_oI}$) has been shown to be effective in solving engineering design and trade-off problems in several diverse industries, including aircraft gas turbines [Law and Antonsson, 1994a], [Law and Antonsson, 1994b], passenger automobile structures [Law and Antonsson, 1996], [Scott and Antonsson, 1996b], [Scott et al., 1997b], and spacecraft reentry aeroshells [Scott et al., 1997a].

The presence of imprecision in engineering design can be illustrated by two simple examples. Figure 1 shows a specification for one performance variable (p_j). As specifications are commonly written, $p_j \geq 250$ km would be represented by the dashed line (the sharp-edged rectangular step), where $\mu_p = 1$ in the acceptable region. However, this crisp specification (or requirement) indicates that two different designs, one with $d_j = 250 - \epsilon$ and $d_j = 250 + \epsilon$ would have completely different acceptabilities, no matter how small ϵ becomes. Thus two designs, indistinguishably different in d_j (as $\epsilon \to 0$), have completely different preferences: one is completely acceptable and one is unacceptable. This situation makes no sense.

Alternatively, the solid line shown in Figure 1 indicates a smooth transition of acceptability of performances from unacceptable ($\mu_p = 0$) to acceptable ($\mu_p = 1$), and thus reflects a more realistic specification. The range over which the transition from unacceptable performance to most desired performance takes place will depend on the particular design

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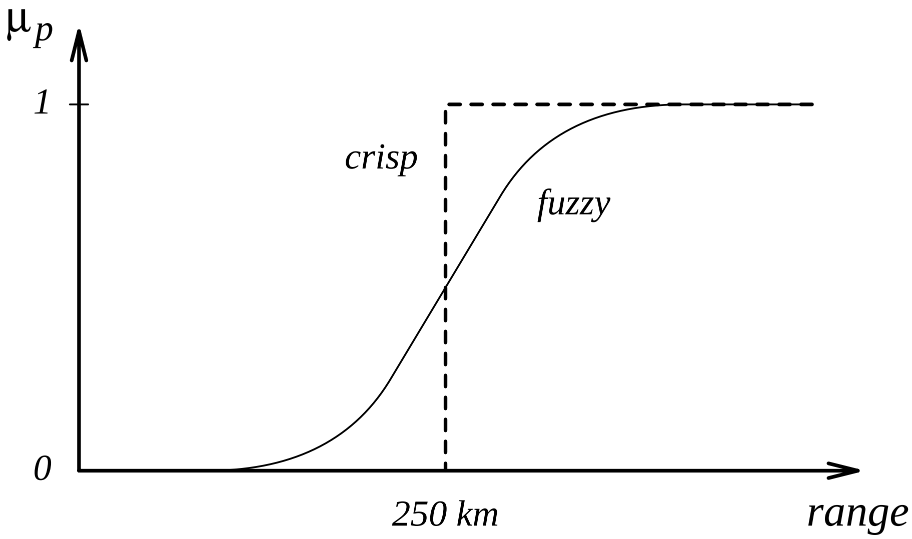

Figure 1: Example imprecise specification.

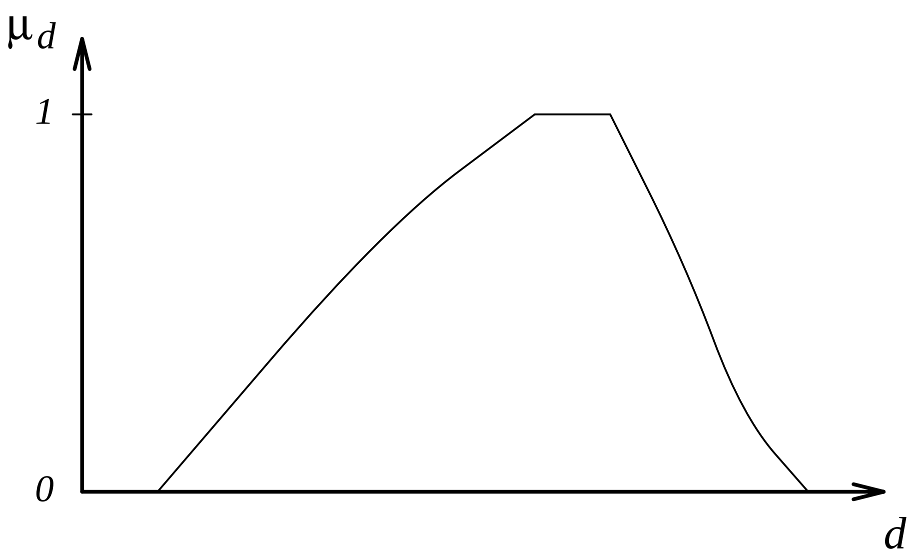

Figure 2: Example imprecise design variable.

problem, and may be more or less steep, and smooth or faceted. For the rare case where a specification is truly crisp, this is a degenerate case of a fuzzy specification, and is easily handled by the M$_o$I.

A second example is shown in Figure 2, where a design remains incompletely refined, but the designer has preferences for using various values of the design variable d. The range indicated with a preference (μ_d) equal to 1 is the range that the designer would most like to use. This preference may arise for objective reasons: cost, space, availability, manufacturability, *etc*., or for subjective reasons: previous experience, judgement, *etc*. The range indicated by $\mu_d > 0$ is the range of values the designer believe are acceptable, at least to some degree. The degree of acceptability for each value of d is indicated by its corresponding value of μ_d, with 0 being the least desirable and 1 the most desirable. In common terms, the range corresponding to $\mu_d = 1$ is the designer's answer to the question: "What would you like?"; the $\mu_d > 0$ range corresponds to "What can you tolerate?" In this way designer and customer preferences are used to indicate imprecision by use of the mathematics of fuzzy sets. Evaluation of the performance of (fuzzy) sets of designs helps the designer by providing measures of possible ranges of performance early in the design process.

To evaluate sets of designs an overall best design variable set must be found, and the concept of "best" must be defined. Unfortunately, performance variables are usually incommensurate (cost, weight, stiffness, power, speed, *etc*.). A traditional approach to combining such incommensurate variables is to use a normalization and a weighting (a weighted sum). This approach requires both a conversion of units (*e.g*., when combining cost and weight and speed into one overall performance measure), and a measure of relative importance of the individual attributes (*e.g*., relative weights).

Instead, incommensurate variables can be combined more usefully using a common trait: designer preference. This means that preference information (μ_d) on the design variables (d_i) and requirement preferences (μ_p) on the the performance variables (p_j) are combined into an overall preference rating (μ_o) for that design variable set ($\vec{d}$). This computation and combination is discussed further below.

Imprecision vs. Uncertainty. Uncertainty, which usually represents uncontrolled stochastic variations with the mathematics of probability, is distinct from imprecision. Uncertainty occurs throughout engineering design, in the form of manufacturing variations, material property variations, *etc*. Including uncertainty in engineering design decision-making can help produce robust designs by assessing the expected size of variations and determining the risk of failure. Many design methods have been developed specifically to address these calculations, including:
Taguchi's method [Byrne and Taguchi, 1986], [Taguchi, 1986], [Kackar, 1985],
probabilistic optimization [Adby and Dempster, 1974], [Arora, 1989], [Papalambros and Wilde, 1988], [Pierre, 1986], [Reklaitis et al., 1983],
weighted sum techniques [Dlesk and Liebman, 1983], [Eschenauer et al., 1990], [Freiheit and Rao, 1988], [Osycska, 1984], [Siddall, 1972], [Steuer, 1986], [Woodson, 1966],
iterative multi-objective design formulations [Ashley, 1992], [Ramaswamy et al., 1991],
and utility theory [Fishburn, 1982], [French, 1988], [Keeney and Raiffa, 1993].
As discussed further below, the M_oI can incorporate probabilistic uncertainties into the design imprecision calculations to produce a measure of the best overall preference in the presence of uncontrolled variations.

AGGREGATION FUNCTIONS

Nearly all formal design methods for representing uncertainty or imprecision utilize one or more functions to aggregate information from multiple attributes. The aggregation calculation performs a trade-off, such that some aspects of a design may contribute more heavily to the combined result than others.

A aggregation function $\mathcal{P}$ is a formalization of the process of trading-off competing design attributes, and should satisfy the restrictions for engineering design proposed in [Otto and Antonsson, 1991]. Aggregation functions can be divided into two classes: compensating and non-compensating. A compensating aggregation function (*e.g.*, *sum*) will produce an overall measure of a design alternative where aspects that perform well can compensate for aspects that perform poorly. For example, a potential customer of a new car may prefer plenty of leg room and good fuel economy, and be willing to let a bit more leg room partially compensate for poor fuel economy when creating an aggregate evaluation of a particular new car. A non-compensating aggregation function (*e.g.*, *min*) will produce an overall measure of a design alternative that is limited by the most poorly performing aspect.

Formalizing the aggregation of attributes permits trade-off strategies that are determined informally or implicitly to be decided rationally and explicitly. A formal trade-off method also permits design decisions to be clearly understood and recorded for later retrieval and examination. When a question regarding a particular design trade-off arises at a later stage in the design process, a formal method can provide a clear and complete picture of how the decision was reached. Moreover, the trade-off can be repeated with revised information, thus confirming or refuting the original decision.

METHOD OF IMPRECISION

The Method of Imprecision (M_oI), introduced in [Wood and Antonsson, 1987] and [Wood and Antonsson, 1989] and reviewed in [Antonsson and Otto, 1995], is a formal (computable) method for representing and manipulating design imprecision (uncertainty in choosing among alternatives) using the mathematics of fuzzy sets. Several groups are currently applying fuzzy methods to engineering design problems: [Scott and Antonsson, 1995], [Scott and Antonsson, 1996a], [Zimmermann and Sebastian, 1993], [Zimmermann and Sebastian, 1994], [Zimmermann and Sebastian, 1995], [Zimmermann, 1996], [Hamburg and Hamburg, 1993], [Müller and Thärigen, 1994], [Chen and Otto, 1995], [Knosala and Pedrycz, 1992], [Posthoff and Schlosser, 1994], [Rao, 1987], [Rao and Dhingra, 1989], [Rao et al., 1990], [Rao and Dhingra, 1991], [Sakawa and Kato, 1995], [Thurston and Carnahan, 1990], [Thurston and Carnahan, 1992].

Imprecision is represented by a range, and a function defined on this range (μ_d), to describe the desirability of (or preference for) particular values, as illustrated in Figure 2. In this way variables whose values are not known precisely can be represented, and can incorporate the designer's experience and judgement into the design evaluation. Non-parametric attributes (material choice, color, style, *etc*.) as well as real-valued attributes (physical dimensions, material properties, performance, cost, *etc*.) can be used.

In the M_oI, constraints can be similarly imprecise, permitting the customer to specify preferences over a range of values, rather than a crisp constraint that may be moved by negotiation later in the design process. Because the method was developed specifically for engineering design, the trade-off combination functions meet the restrictions discussed in [Otto and Antonsson, 1991]. A choice of two basic combination functions is available to aggregate the preferences for the attributes of the design: the (non-compensating) *min* and a (compensating) product of powers. A family of aggregation functions suitable for engineering design decision-making are presented in [Scott and Antonsson, 1995], [Scott and Antonsson, 1996a]. These functions span from non-compensating to partially compensating to fully compensating to supercompensating.

Because the M_oI does not require all attributes to be aggregated into one evaluation metric, evaluations of the various aspects of a design can be made in a hierarchy. For example, safety margin might be traded-off in a non-compensating way among several parts of the design that are subject to loading, and weight and cost might be traded-off in a compensating way. The results of those two trade-offs then might be traded-off with a non-compensating combination function. Because importance weighting can be readily applied, the relative weight of each aspect of the decision can be incorporated into the hierarchy [Law and Antonsson, 1995].

Finally, stochastic uncertainty (such as uncontrolled manufacturing variations) and possibilistic uncertainty and necessity (such as post-manufacturing tuning adjustments) can be incorporated into the design decision-making by utilizing well known expectation calculations [Otto and Antonsson, 1994].

Utility theory and the M_oI are strongly similar when there is only one goal, and a compensating strategy is used [Otto and Antonsson, 1993b]. When goals are traded-off in a non-compensating manner, and without considering importance weightings, the M_oI reduces to a convolution of the constraints and goals used in fuzzy sets for decision-making [Bellman and Zadeh, 1970].

IMPRECISION CALCULATIONS

After specifying design preferences μ_{d_i} on the design variables and specifications and requirements μ_{p_j} on the performance variables, and identifying a (possibly hierarchical) design trade-off strategy, the next step in applying the M_oI is to determine the induced values of μ_{d_i} on the performance variables (design preferences mapped onto the performances), given by the extension principle [Zadeh, 1965]:

$$\mu_d(\vec{p}) = \sup_{\vec{d}:\vec{p}=\vec{f}(\vec{d})} \left[\mu_d(\vec{d})\right] \tag{1}$$

A simple one-dimensional example of Zadeh's extension principle is shown in Figure 3. The performance p achieved for each value of the design variable d is given by the function f, which is a curve in this simple example.[1] Just as the performance p for each value of d can be found, the corresponding $\mu_d(d)$ can be mapped onto p, producing $\mu_d(p)$: the design preference mapped onto the performance space (as illustrated by the dashed lines in Figure 3). In this simple example, with only one d and one p, the function that relates the two is a curve. For more realistic design problems, each p will be a function of many d's, and each function f will be a surface or hyper-surface.

An algorithm to compute Zadeh's extension principle (and thus to calculate $\mu_d(\vec{p})$) is the *Level Interval Algorithm* (LIA), first proposed by [Dong and Wong, 1987] as the "Fuzzy Weighted Average" algorithm and also called the "Vertex Method", and extended by [Wood et al., 1992], [Otto et al., 1993], [Law and Antonsson, 1994a], and [Law, 1996].

Once the imprecision on each design variable ($\mu_d(\vec{d})$) is induced onto the performance variables, the induced preferences are combined with the specifications and requirements ($\mu_p(\vec{p})$) to obtain an overall preference ($\mu_o(\vec{p})$). The point (or points) with the highest preference correspond to the performance of the overall most preferred design(s). The design problem is to find the corresponding set of design variables ($\mu_d(\vec{d^*})$) that produce the maximum overall preference (μ_o^*). In the typical engineering design case, where the inverse mapping ($\vec{f}^{-1}$) doesn't exist, $\mu_o(\vec{d})$ can still be obtained point by point [Law and Antonsson, 1994a].

NOISE

Having formulated the overall preference function (by inducing the designers preference μ_d on the performance variables, and combining the induced preferences with the performance specifications μ_p), there may still be uncertainties (noises) that confound the search for the design variable set which provides the highest overall preference despite variations. This is similar to Taguchi's measure of "quality" [Taguchi, 1986], except his measure applies to only a single performance variable; the M_oI applies to many performance variables [Otto and Antonsson, 1993a].

Three types of noise (uncontrolled variations) are observed [Taguchi, 1986]: external, internal, and variational. External noise is due to environmental fluctuations, such as operating temperatures, humidities, *etc*. Internal noise is inherent in the design, such as wear, storage deterioration of materials, *etc*. Variational noise arises from variations in the supplied

[1] Note that here f is non-linear. Non-monotonic and discrete functions can also be used [Wood et al., 1992], [Otto et al., 1993].

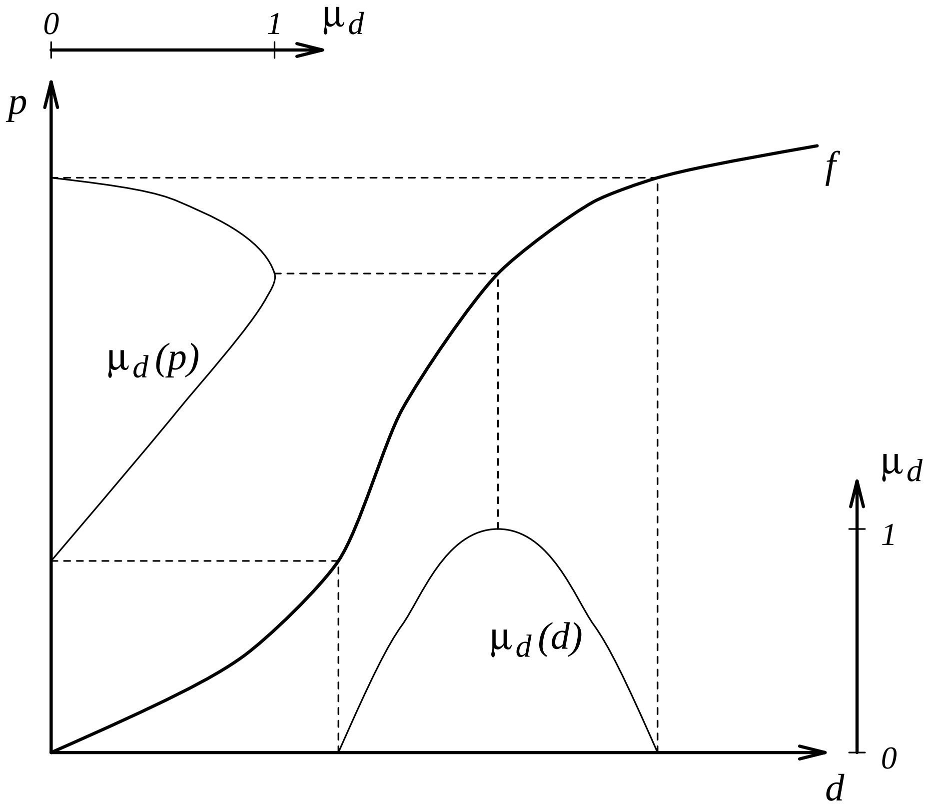

Figure 3: Zadeh's extension principle.

materials and manufacturing processes. A modeling scheme for tolerances on uncontrolled variations and methods for selecting an overall best design variable set in the presence of such variations is introduced in [Otto and Antonsson, 1994].

In addition to tolerances (in which a variable can take on any one value) a variable may need to satisfy *every* value of an interval. Such variables are termed *necessary variables* [Ward, 1989]. The mathematics of necessity was introduced to engineering design in [Ward et al., 1990] using interval mathematics. The concept has been extended to fuzzy sets and to multiple uncertainty forms [Otto and Antonsson, 1994].

In an engineering design problem, noise is typically characterized by *noise variables*, $n_1, \ldots, n_k, \ldots, n_q$. A noise variable n_k might be the possible positioning of an operator switch, and so the alternatives may be discrete. Alternatively, n_k might be a value of a manufacturing error on a design variable, and so the noise variable space (NVS) may be continuous.

Noise is included in design decision making to minimize the effects of uncontrolled variations by proper selection of the nominal value of each imprecise (design) variable. The most preferred value for each design variable is determined by weighting the preference of the design variable set by its probability of occurring through the probabilistic uncertainty. This implies that given a probability space, the preference of a point d in the design variable space is defined by:

$$\mu(d) = \int_{NVS} \mu(d,n)\; dPr \tag{2}$$

where $\mu(d,n) = \mathcal{P}(\mu_1, \ldots, \mu_N)$. Thus, $\mu(d)$ is the probabilistic expectation of $\mu(d,n)$ across the probability space with respect to the probability measure Pr. The most preferred design configuration d^* is the one which maximizes the expected preference of Equation 2 across the design variables.

Taguchi's method is an experimental approximation of the integral in Equation 2 [Otto and Antonsson, 1993a] to evaluate "quality": the mean of a single performance variable p (the S/N ratio). In the M$_\text{O}$I, Equation 2 is used to compute the preference of many design and performance variables, and is thus a generalization of Taguchi's method.

Finally, some variables are adjusted (either during or after manufacture) to optimize the operation of each individual device. Needle valves in carburetors, trim potentiometers in electronic devices, *etc.*, are common examples of tuning variables (introduced in [Otto and Antonsson, 1993c]), and are characterized by the ability of the tuning variable to compensate for noise. The space spanned by the tuning variables is tuning variable space (TVS).

For problems with multiple uncertainty forms, the *precedence relation* among the variables must be made explicit

[Otto and Antonsson, 1994]: design variables on the outside, and tuning variables on the inside (relative to the confounding noise variables). This is discussed in [Otto and Antonsson, 1993a], [Otto and Antonsson, 1993c].

Thus, with multiple forms of uncertainty, the most preferred design variable values d^* are:

$$\mu(d^*) = \sup\left\{ \int_{Pr(\delta d)} \sup\left\{\mu(d, \delta d, t) \mid t \in \text{TVS}\right\} dPr(\delta d); \middle| \, d \in \text{DVS} \right\} \tag{3}$$

where δd are the manufacturing errors and t are the tuning variables. The maximization of the tuning variable selection occurs inside the integral across the manufacturing errors, and the maximization of the design variable selection occurs outside the integral across the manufacturing errors. For a general design problem, the evaluation order of the maximizations, minimizations, and integrals will depend on the precedence relation among the variables, determined by the structure of the problem. An example of precedence relation ordering in an engineering design problem is shown in [Otto and Antonsson, 1993c].

MAPPING DESIGN IMPRECISION

In implementing the Method of Imprecision, a key step is mapping design preference μ_d from the n-dimensional DVS to the q-dimensional PVS. Here computational complexity arises from two distinct sources. If the individual design preferences $\mu_{d_1}, ..., \mu_{d_n}$ are to be combined with a non-compensating aggregation function $\mathcal{P}_{\min}$, the combination of design preferences can be easily computed with a method such as the Level Interval Algorithm (LIA) [Dong and Wong, 1987]. For aggregation functions other than $\mathcal{P}_{\min}$, more specialized techniques are needed. Furthermore, when the evaluation of the mappings f_j from the DVS to the PVS is expensive, it is unrealistic to exhaustively search a DVS of more than a few dimensions.

The key limitation of the LIA, that it requires monotonicity, stems from the assumption that the extreme values of f_j will occur at the corner points of $D^d_{\alpha_k}$, the n-cube which is the α-cut at α_k in the DVS. The algorithm may thus be improved by relaxing this assumption [Mathai and Cronin, 1995]. The extended problem is to find:

$$\begin{aligned} {p_j}^{\alpha_k}_{\min} &= \min\{p_j = f_j(\vec{d}) \mid \vec{d} \in D^d_{\alpha_k}\} \\ {p_j}^{\alpha_k}_{\max} &= \max\{p_j = f_j(\vec{d}) \mid \vec{d} \in D^d_{\alpha_k}\}. \end{aligned} \tag{4}$$

Finding extrema within a subset of the DVS is a constrained optimization problem.

In choosing an optimization technique, a trade-off must be made between computational cost and robustness (*i.e.*, the ability to find the correct global extremum for various starting conditions). Traditional calculus-based optimization methods converge in relatively few function evaluations but seek only local minima. Randomized search methods such as genetic algorithms offer greater robustness [Goldberg, 1989] but require more function evaluations. The computational implementation employed by the M_OI uses Powell's method, a calculus-based optimization algorithm [Adby and Dempster, 1974]. An important feature for a practical computational tool is a means to trade-off the number of function evaluations against accuracy and reliability. Such an adjustment enables the designer to use the same program to obtain quick estimates as well as precise evaluations. This is implemented as a user-specified fractional precision that defines termination criteria for the optimization algorithm. Extensions to the LIA are presented in detail in [Law, 1996].

The second difficulty can be partially overcome by selectively approximating $\vec{f}$ as a simple function $\vec{f}'$ over D^d_ϵ (the α-cut at infinitesimal $\alpha = \epsilon$, which represents all designs under consideration). A linear approximation is not the only choice, but higher order approximations introduce additional complexity, both in the shape of the level sets mapped onto the PVS and in the computational algorithm, that is not often justified [Law, 1996].

The approach used in the M_OI is similar to *response surface methods* [Montgomery, 1991], which seek to optimize a response that is influenced by several variables. The function f_j is modeled over the search space D^d_ϵ. The linear approximations $f'_1, ..., f'_q$ are obtained using techniques adapted from statistical design of experiments. These techniques rely on orthogonal arrays, which specify an efficient, independent set of points at which the function is evaluated. Orthogonal arrays are widely used not only for statistical design of experiments but also for the Taguchi Method or Robust Design methodology [Peace, 1993], [Phadke, 1989] and their direct application to engineering design is not new [Chi and Bloebaum, 1995], [Korngold and Gabriele, 1995].

CONCLUSION

Imprecision and uncertainty occur throughout the engineering design process. Many methods for incorporating uncertainty (*e.g.*, utility theory, probability methods, Taguchi's method, *etc.*) are in common use, however, methods to represent imprecision in engineering design are only now under development. The Method of Imprecision (M_oI) is a formal method for incorporating the natural level of imprecision that occurs throughout the engineering design process, and can include: many incommensurate aspects of a design, imprecise constraints, compensating and non-compensating trade-offs, hierarchical trade-offs, importance weightings, and judgement and experience. Uncontrolled variations (noise) can also be incorporated so that the design with the greatest overall preference and most robustness to the noise can be found.

By providing the designer and customer with a technique to specify preferences on design and performance variables, design communication will evolve from individual "point" designs to (fuzzy) sets of designs. Since a range of possible design variable values can be released to downstream design processes earlier than a completed individual design, the MoI facilitates (fuzzy) set-based concurrent design.

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