This is an amendment/correction to *Geometric Modeling* by Michael E. Mortenson, 1997, second edition, John Wiley and Sons, Pages 114 and 115.

Some of his nomenclature is ambiguous. Here it is presented in clearer form, by making a distinction between k (the order of the intermediate blending function being calculated) and K (the final k, which is the order of the whole B-spline). A distinction is also made between the index i on blending functions $N_{i,k}$ and the index j on knot values t_j. Additions to the text are in italics.

Note that the example that begins at the bottom of Page 115 is for a *final* $K = 1$. The example that begins near the middle of Page 117 is for a *final* $K = 2$. The example that begins in the middle of Page 118 is for a *final* $K = 3$. Note that these three examples do not build on each other. $N_{i,1}$ and $N_{i,2}$ get recomputed (with $K = 3$) before forming $N_{i,3}$ in the example beginning in the middle of Page 118 where $K = 3$.

Michael J. Scott, 12-May-1997; Erik K. Antonsson, 7-May-1998

. . .
The nonrational form is given by

$$\mathbf{p}(u) = \sum_{i=0}^{n} \mathbf{p}_i N_{i,K}(u) \tag{5.1}$$

Comparing Equation 5.1 to Equation 5.2 . . .
. . .
For a B-spline curve, a parameter K controls the degree of the basis polynomials, and it is usually independent of the number of control points, except as it is limited by Equation 5.6, following.
. . .
The nonuniform B-spline blending functions are defined recursively by the following expressions:

$$\begin{aligned} N_{i,1}(u) &= 1 \quad \text{if } t_i \le u < t_{i+1} \\ &= 0 \quad \text{otherwise} \end{aligned} \tag{5.2}$$

and

$$N_{i,k}(u) = \frac{(u - t_i)N_{i,k-1}(u)}{t_{i+k-1} - t_i} + \frac{(t_{i+k} - u)N_{i+1,k-1}(u)}{t_{i+k} - t_{i+1}} \tag{5.3}$$

for integer values of k:

$$k = 2, \ldots, K$$

where K, *the order of the B-spline,* controls the degree $(K-1)$ of the resulting polynomial in u and also the continuity of the curve. The t_j are called *knot values,* and a set of knot values comprises a *knot vector*. They relate the parametric variable u to the $\mathbf{p}_i$ control points *where* $i = 0, \ldots, n$. For an open nonuniform curve that interpolates the endpoints, the t_j are *calculated once using* K:

$$\begin{aligned} t_j &= 0 && \text{if } j < K \\ t_j &= j - K + 1 && \text{if } K \le j \le n \\ t_j &= n - K + 2 && \text{if } j > n \end{aligned} \tag{5.4}$$

for integer values of j:

$$j = 0, \ldots, n + K$$

Note that once the t_j are calculated for K, they are used to compute $N_{i,k}$ for all $k = 1, \ldots, K$. The index j on knot values t_j ranges from 0 to $(n+K)$. The knot values t_j themselves take on values from 0 to $(n - K + 2)$. The index i on blending function $N_{i,k}$ ranges from 0 to n. There are always $n + 1$ blending functions for each k.

The parameters determining the number of control points, knots, and the degree of the polynomial are related by

$$n + K + 1 = T \tag{5.5}$$

where T is the number of knots. For nonuniform and open B-Spline curves, the knot vector $\mathbf{T}$ is characterized by

$$\mathbf{T} = \{\alpha, \alpha, \ldots, \alpha, t_K, \ldots, t_{T-K-1}, \beta, \beta, \ldots, \beta\}$$

where end knots α and β are repeated with multiplicity K.

If the entire curve is parameterized over the unit interval, then, for most situations $\alpha = 0$ and $\beta = 1$. However, if we assign nondecreasing integer values of the parameter to the knots, then $\alpha = 0$ and $\beta = n - K + 2$.

Spacing the knots at equal intervals ...

...

So, we will define the range of the parametric variable u to be

$$0 \leq u \leq n - K + 2 \tag{5.6}$$

Because the denominators in Equation 5.2 can be zero, we must define $0/0 = 0$.